AMERICAN UINVERSITY OF BEIRUT
FACULTY OF ENGINEERING AND ARCHITECTURE
EECE 460 Control Systems
Spring 2004-2005
Quiz II Solution
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Name:

1.5 hours.
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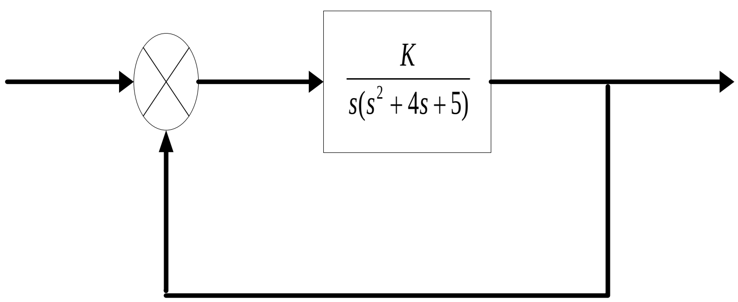
Total of 100 points
Open Book Exam, 2 pages
YOU MUST RETURN THIS EXAM WITH YOUR ANSWER BOOKLET

Problem 1 (30 points):

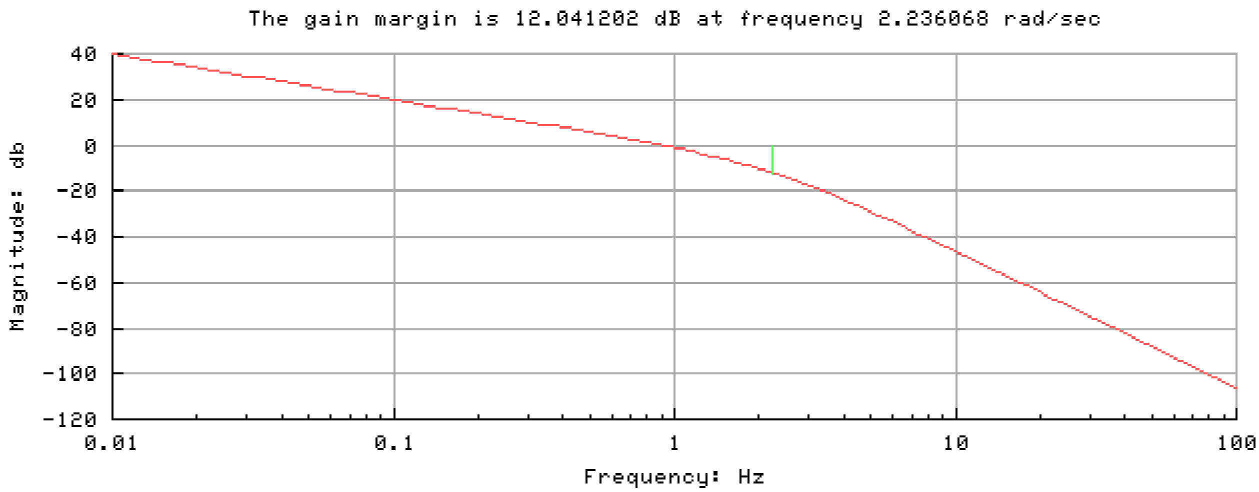
The following process transfer function represents a SISO LTI system. Assume that the closed loop system has unity feedback.

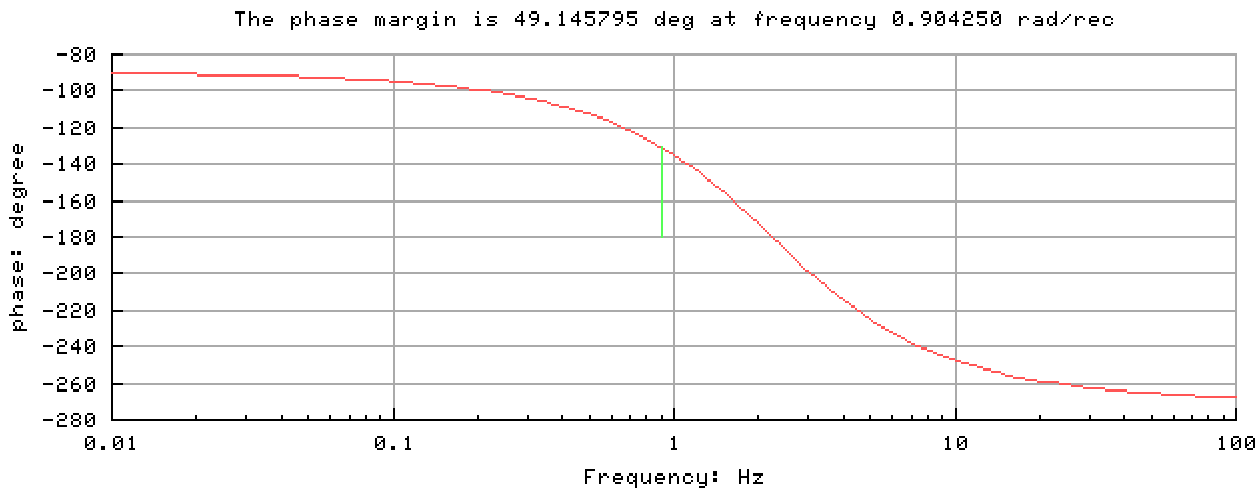


a) If the system input was 2Sin(3t), compute the frequency response. **C.L.T.F** (**15 pts)**

Ysst =0.3*sin(3*t + 2.775)

b) Estimate based on Bode plots the system Phase and gain Margins. **O.L.T.F.(15 pts)**





Problem 2 (30 points):

A continuous system is modeled by the following state equations:

$$\dot{x}(t) = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

and output Y(t)=X1(t); u(t) is the input of the system.

a) Find the corresponding Transfer function. State any assumptions you made. (**10 pts)**

Zero I.C. $TF = \dfrac{1}{s^2 - 1}$

b) Find the Controllable Canonical Form State model with a state vector Z(t). (**8 pts)**

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

c) Denote the relationship between X(t) and Z(t) by the linear transformation matrix T, where Z(t)=TX(t). Determine T. (**7 pts)** $T = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

d) Determine the relationship between the Matrices (i.e. A,B,C) associated with the 2 different State Models in X(t)& Z(t). (**5 pts)**

$$A = T^{-1} A_c T$$
$$B = T^{-1} B_c$$
$$C = T^{-1} C_c$$

Problem 3 (40 points):

The following process is modeled by a state model that represents a SISO LTI system.

$$\dot{x}(t) = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

and output Y(t)=X1(t); u(t) is the input of the system.

a) Design if possible an estimator for all internal states. (**15 pts)**

Check that system is fully observable. Design states to be 10 times faster than (-2 and 3) in part c.

b) Design if possible a pole placement scheme using the estimates of the states multiplied by constant gains as feedback to force the closed loop system to have desired poles located at -2 and -3. (**15 pts)**

Check that system is fully controllable

$$s_1 = -2 \text{ and } s_2 = -3 \text{ then } K = \begin{bmatrix} 12 & 5 \end{bmatrix}$$

c) What are the achieved specs in case of a step response (Natural frequency, Damping ratio, and therefore Settle time and overshoot). (**10 pts)**

$$s^2 + 5s + 6 = s^2 + 2\zeta w_n + w_n^2$$

$$w_n = 2.45$$

$$\zeta = 1.02 > 1 \text{ hence overdamped}$$

Since overdamped==> then there is no Max overshoot

$$t_s = \frac{4}{\zeta w_n} = 1.6 \text{ (2\% criterion)}$$